**ANOVA (For any type)**

* Observations are normally distributed populations with equal variances and different means
* Basic assumption: The natural variation within each factor is the same.
* Homogeneity of variance (same variances) applies to the variation ( within/between) each factor (treatment or combination).
* Independent samples, A factor is being tested at more than one level
* Similar to a test for equality of 2 or more POPULATION means (not sample means)
* The statistic used in **ANOVA**? F statistic, as it is comparing variances between two samples.
* p-value = 0.05 rejects a null hypothesis in an ANOVA table
* Method of analyzing designs when one or 2 factors are to be tested
* Usual test for experiments involving several treatments (factors, independent variables) at various levels
* Always more than one level for each factor, one way ANOVA is one factor but with more than 1 level
* Inference about a population using a single sample mean is determined using either Z or t.

One-way ANOVA:

* One dependent variables,
* ONE independent variable (factor) (2 population means)
* Samples independents
* NOT ONE LEVEL, factors should have more than 1 level

ANOVA: Most accurate method for quantifying repeatability and repoducibility. Allows the variability of the interaction between the appraisers and the parts to be determined.

Use to:

* For significance testing is equality of sample means can be tested by comparing sample variance
* Perform test on the variance of each population, not in the means, it is an indirect way for finding differences in the means.
* Use software for performing ANOVA, you visualize which group does not overlap with the rest.
* When you don’t have charts, use the means, and the mean (or means) that is different would be your different population
* Identifying differences in the means of two or more populations.

The components of variance in ANOVA will determine:

* The extent of contribution by each source of variance
* How much of the variance is due to differences in treatment means
* How much of the variance is due to experimental error

**1 WAY ANOVA: Resulsts from 4 Machines, or 4 Different additives**

|  |  |  |
| --- | --- | --- |
| **A** | **B** | **C** |
| 4 | 2 | -3 |
| 8 | 0 | 1 |
| 5 | 1 | -2 |
| 7 | 2 | -1 |
| 6 | 4 | 0 |

**3 Machines= df = 2 in Between groups**

**15 treatments 3x(5-1) df = 12 in Within groups**

**Or could it be 3 additives tested in 15 runners**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **SUMMARY** |  | |  |  |  |  |  |
| ***Groups*** | ***Count*** | | ***Sum*** | ***Average*** | ***Variance*** |  |  |
| **Column 1** | 5 | | 30 | 6 | 2.5 |  |  |
| **Column 2** | 5 | | 9 | 1.8 | 2.2 |  |  |
| **Column 3** | 5 | | -5 | -1 | 2.5 |  |  |
|  |  | |  |  |  |  |  |
| **ANOVA** |  | |  |  |  |  |  |
| ***Source of Variation*** | ***SS*** | | ***df*** | ***MS*** | ***F*** | ***P-value*** | ***F crit*** |
| **Between Groups** | | 124.133 | 2 | 62.07 | 25.86 | 0.00004 | 3.89 |
| **Within Groups** | | 28.8 | 12 | 2.4 |  |  |  |
|  | |  |  |  |  |  |  |
| **Total** | | 152.93 | 14 |  |  |  |  |

Critical F value for the Between source: F 0.05, 2 , 12 = **3. 88**

We reject Ho hypothesis 25 > 3 (all means are **NOT** equal)

The term “between” in the source table refers to the additives (or the machines)

The term “Within” refers **to the error term**. The Within term describes the variation present among the runners.

A study was made on the effects of several health additives for a number or runners. The One-Way **ANOVA** results are

4 Additives and 19 runners

Table

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The term “between” in the source table refers to the additives,

The term “Within” refers to the error term. The Within term describes the variation present among the runners.

Critical F value for the Between source: F 0.05, 3 , 15 = **3. 28** F Not significant Don’t reject Ho (all means are equal)1

The MS (Mean square) is 55/ 3 = 18.3 450 / 15 = 30. **If it were Error we calculate the MS** The total sum of squares is not divided by the total degrees of freedom, therefore:

* MS has only 2 values = **18.3 and 30.0**
* Number of additive is given by the Between term = 3+1 = **4**
* Number of runners , Total DF 18 + 1 = **19**

**2 WAY ANOVA Materials vrs Machines**

Diagram, schematic

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The missing critical values assuming 95% confidence are F 0.05,2 , 10 = **4.10** F 0.05, 2, 10 =**4. 10**

In both cases null hypothesis is rejected ( F critical < F calculated) hypothesis that all materials and machines are equal is rejected

The appropriate ratio for testing the null hypothesis of no treatment effect is:

F = MST Mean Square Treatment Mean variation between treatments MS Treatments SSx / DFx SSx DF error

MSE Mean Square Error Mean variation within treatments MS residuals SS error / DF error SS error DF x

SS = Sum of squares

When considering qualitative and quantitative factors in the same designed experiment:

The sum of squares for the qualitative factors can still be calculated even though no numerical scale can be attached to the levels.

Results from experiments may be numerical or not

The fundamental equation of ANOVA (Analysis of variance) Total SS = SST + SSE 14 = ( 2 +2 )+ 10

Total sum of squares of deviations from the grand mean = sum of squares of deviation within treatments + sum of squares of deviations among treatments means and the grand mean

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A study conducted to determine the effect of pressure on the thickness of metal stampings had these results:**   |  |  |  |  | | --- | --- | --- | --- | | Treatments | Pressure Low | Pressure Medium | Pressure High | | Average Thickness: | High | Medium | Low | | Variance: | Medium | Low | High | | Samples (n): | 30 | 30 | 30 |   **What are the values for the between treatment (BT) and within treatment (WT) degrees of freedom?**  **BT = 3-1 = 2 WT = 90-3 = 87** |

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Assume 3 populations A, B and C

N = 15 DF = n-1 = 14

**X =** 30

**CRUDE Sum of squares X2** = 222

**Correction for mean CM** =  **(  X ) 2 / N** =(30) 2 / 15 = 60 **(correction factor)**

**Total Sum of squares SS X2  - CM = 222-60 = 162 (Corrected SS)**

Variance = Total sum of squares / total DF = 162 / 13 = 12.46

** (TCM)** = 197.2

**SST** = ** (TCM)** **– CM** = 197.2 – 60 = 137.2

**SSE = Total SS – SST** = 162 - 137.2 = 24.8

The complet ANOVA table is

Table

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F(33.2) exceeds the critical value of F, null hypothesis is rejected

It will be seen that the 2-way analysis procedure is an extension of the patterns described in the 1-way analysis. Recall that a 1-Way ANOVA has 2 componets of variance: Treatments and experimental error (may be referred to as columns and error or rows and error).

In the 2-Way ANOVA there are 3 componets of variance:

Factor A Treatments

Factor B treatments

Experiemtnal error (may referred to as columns, rows and error)

ANOVA can be extended with a determination of the COV (Components of Variance). The COV table uses the MS (Mean Square), F and F(alpha) columns from the previous ANOVA table and adds columns for EMS (Expected Mean Square), Variance, Adjusted Variance and Percent Contribution to design data variation.

ANOVA for A x B factorial experiment

Factor A with a levels / Factor B with b levels

Table

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ANOVA for Randomized Block Design

Randomized Block Design implies the presence of 2 independent variables, “block” and “treatments”

The total sum of square of the response measuments can be partitioned into 3 parts: Sum of the squares for the blocks, treatments and error. The analysis of a randomized block design is of less complexity than an A x B factorial experiment

Table

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